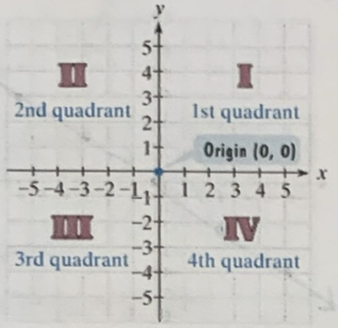
Name: Key Class Period: ____ Janik208 N

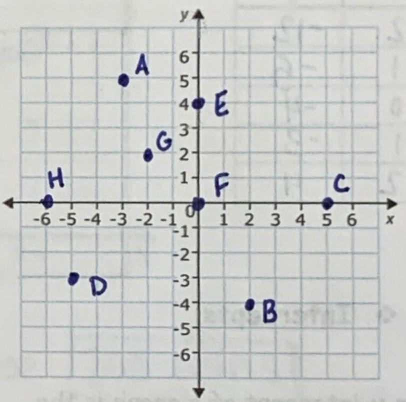
Unit 1 – Functions and Their Graphs
Chapter 1 -Review of Functions and Their Graphs
Notes Packet



This graph has two names. It can be called the rectangular Coordinate System or the Cartesian Coordinate System. The horizontal number line is the x-axis and the vertical number line is the y-axis. The point that occurs where the axes cross is called the origin. (-5 ,3) and (3, -5) are examples of coordinates or ordered pairs. The axes divide the plane into four quarters, called quadrants. The points that are located directly on the axes are not in any of the quadrants.

❖ Plotting Points

Plot the points and state the quadrant.



A(-3,5)	B(2, -4)	C(5, 0)	D(-5, -3)
II	IV	x-axis	III
E(0, 4)	F(0, 0)	G(-2, 2)	H(-6, 0)
y-axis	origin	II	x-axis

-Part 1 HW

❖ Graphs of Equations

A relationship between two quantities can be expressed as an equation with two variables such as $y=4-x^2$. A solution of an equation with two variables, such as x and y, is written as an ordered pair. We can also say that the solution satisfies the equation. **Example:** Is (3, -5) a *solution* to y= 4 – x^2? How do you know?

x y -5 3

$-5=4-(3)^2$

$-5=4-9$

$-5=-5$ ✓

Because both sides are equal, the ordered pair (3, -5) satisfies the equation.

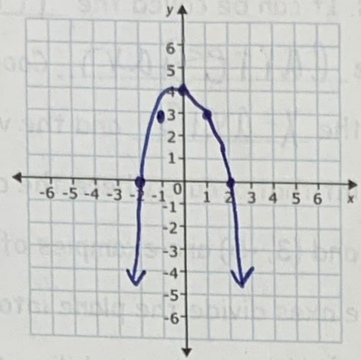
The graph of an equation with two variables is the set of all points whose coordinates satisfy the equation.

Examples: *Without using a calculator*, graph the following equations. Eventually, we will know how to graph these equations using different methods. For now, we will use the point plotting method. Choose 5 points, substitute in each number for x, use order of operations, and solve for y.

*A common question I always get is, what points do I choose? Choose 5 "Smart" points !

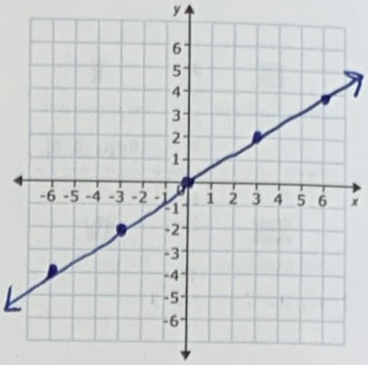
$y = 4 - x^2$

x	y
-2	0
-1	3
0	4
1	3
2	0



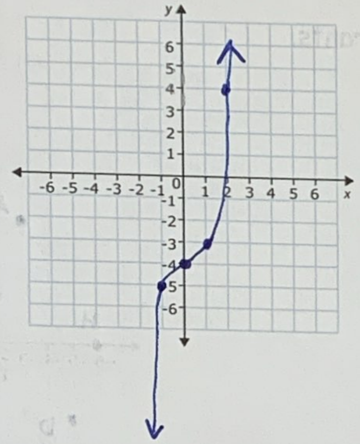
$y = \frac{2}{3}x$ → pick points divisible by 3!

x	y
-6	-4
-3	-2
0	0
3	2
6	4



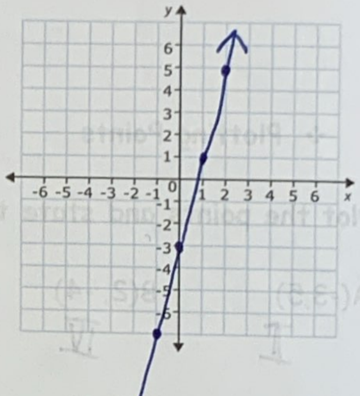
$y = x^3 - 4$

x	y
-2	-12
-1	-5
0	-4
1	-3
2	4



$y = 4x - 3$

x	y
-2	-11
-1	-7
0	-3
1	1
2	5

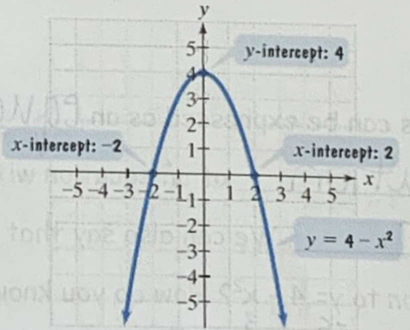


-Part2 HW

❖ **Intercepts**

An x-intercept of a graph is the x-coordinate of a point where the graph intersects the x-axis. The y-coordinate corresponding to an x-intercept is always 0.

(# , 0)



An y-intercept of a graph is the y-coordinate of a point where the graph intersects the y-axis. The x-coordinate corresponding to an y-intercept is always 0.

(0 , #)

Example: Find the x-intercept and the y-intercept of the equation $y = 4x - 3$.

Find the x-intercept: Set y = 0

$0 = 4x - 3$

$+3 \quad +3$

$\frac{3}{4} = \frac{4x}{4}$

$\boxed{(3/4, 0)}$

$x = 3/4$

Find the y-intercept: Set x = 0

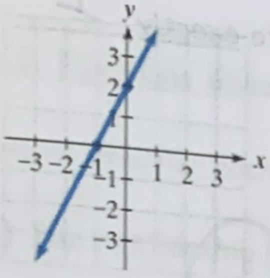
$y = 4(0) - 3$

$y = -3$

$\boxed{(0, -3)}$

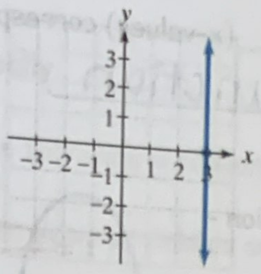
amples: Identify the x and y intercepts. Be careful... there may be more than one!

a.



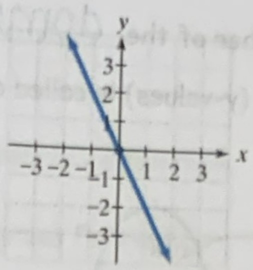
x-int(s):	(-1,0)
y-int(s):	(0,2)

b.



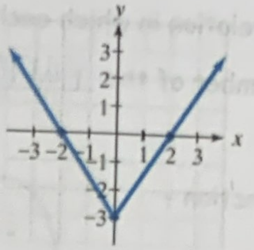
x-int(s):	(3,0)
y-int(s):	none

c.



x-int(s):	(0,0)
y-int(s):	(0,0)

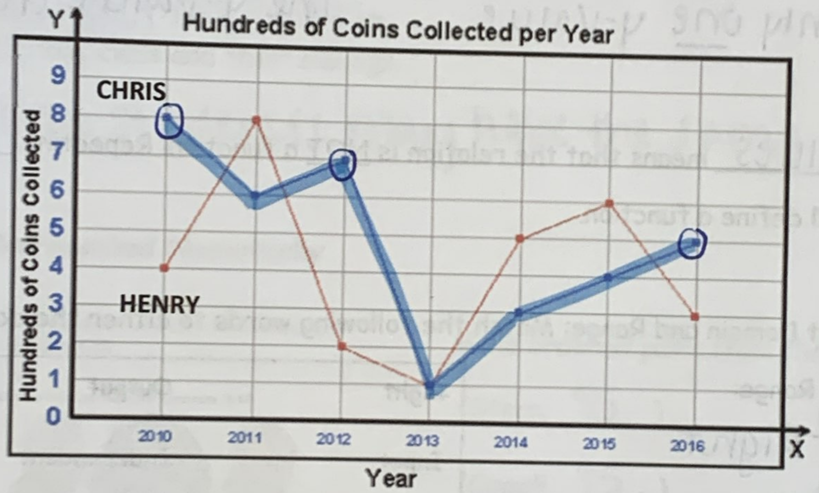
d.



x-int(s):	(-2,0),(2,0)
y-int(s):	(0,-3)

-Part 3 HW

Reading a graph:



1.) What was the difference in coins collected by Chris and Henry in 2015? 200 coins

400 600

2.) How many months did Chris collect more coins than Henry? 3years = 36 months

3.) In the years 2010 and 2011, how many coins did the students collect together?

2010: 400+800=1200

2011: 600+800=1400

1200+1400 = 2600 coins

-Part 4 HW

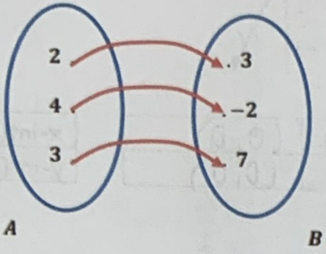
❖ **Relations**

A relation is any set of ordered pairs. A relation is a set of inputs and outputs that are related in some way. The set of all inputs (X-values) of the ordered pairs is called the domain of the relation and the set of all outputs (y-values) in the ordered pairs is called the range of the relation.

❖ Functions

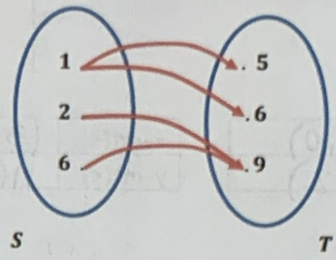
A relation in which each member of the domain (x-values) corresponds to exactly 1 member of the range (y-values) is called a function.

Function -



Every x is paired with only one y-value

Relation -



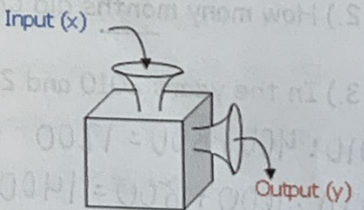
Each x has more than one y-value (repeating x's)

*Repeating X-Values means that the relation is NOT a function! Repeating Y-Values are okay and may still define a function.

Characteristics about Domain and Range: Match the following words to either the domain or range.

Domain:	Range:
-Left	-Right
-Input	-Output
-X-values	-Y-values
-Independent	-Dependent

~~Right~~	~~Output~~	~~Left~~
~~Input~~	Independent	~~Y-Values~~
Dependent	~~X-Values~~	



Functions can be represented in four ways:

1.) Verbally - a sentence that states how the input variable is related to the output variable

2.) Numerically - tables, a list of ordered pairs, or mapping

3.) Graphically - horizontal axis - input values, vertical axis - output values

4.) Algebraically - equation with two variables you are stuck, make ordered pairs corresponding to the data!

Type 1: Functions Given Verbally

Determine whether each of the following situations represents a function.

a.) The relationship between degrees Celsius and degrees Fahrenheit

Function - every degree C corresponds to one degree F

b.) On the moon, items weigh 1/6 their weight on Earth. The relationship between an item's weight on Earth and its weight on the moon.

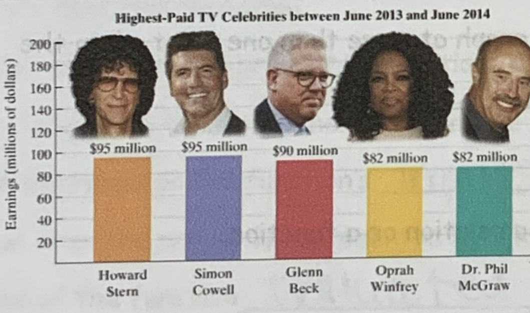
Function - every weight on the moon corresponds to one weight on Earth.

c.) The students in this class and their siblings.

Relation - some students may have the same # of siblings

Type 2: Functions Represented Numerically

Forbes magazine published a list of the highest-paid TV celebrities between June 2013 and June 2014. The results are shown in **Figure 1.13.**



Make ordered pairs from the graph on the left:

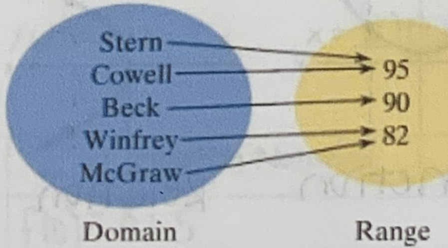
(Stern, 95)

(Winfrey, 82)

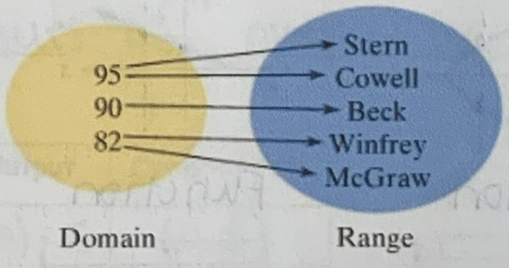
(Cowell, 95)

(McGraw, 82)

(Beck, 90)



Function



Relation only

(95, S)

(95, C)

(90, B)

(82, W)

(82, M)

Examples: Determine whether each relation is a function.

$\{(1,6),(2,6)(3,8),(4,9)\}$

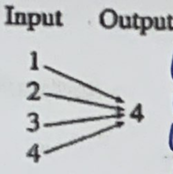
Function

$\{(6,1),(6,2),(8,3),(9,4)\}$

Relation

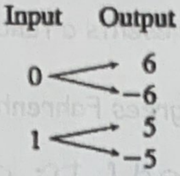
$\{(1,2),(3,4),(6,5),(8,5)\}$

Function



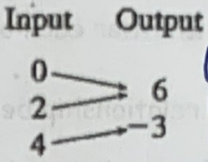
(1,4)
(2,4)
(3,4)
(4,4)

Function



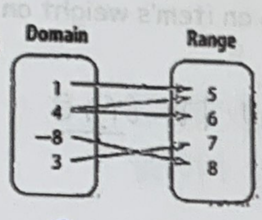
(0,6)
(0,-6)
(1,5)
(1,-5)

Relation



(0,6)
(2,6)
(4,-3)

Function



(1,5)
(4,5)
(4,6)
(-8,8)
(3,7)

Relation

Domain	Range
4	6
−5	3
6	−3
−5	5

Relation

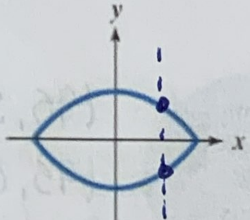
Domain	Range
−4	2
3	−5
4	2
9	−7
−3	−5

Function

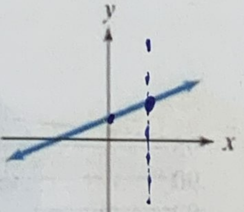
Type 3: Functions Represented Graphically

Not every graph you see represents a function. Remember, each value of X can only be paired with one y-value. On a graph, you can see and determine this by using the vertical line test. The VLT says that if any vertical line intersects the graph at more than one point, then the graph does not define y as a function of x.

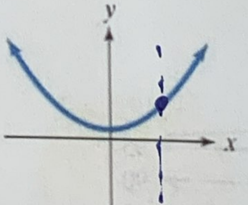
Examples: Using the VLT, determine if each graph represents a relation or a function.



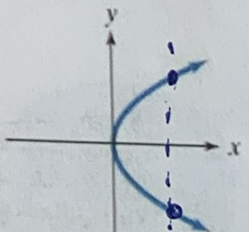
Relation



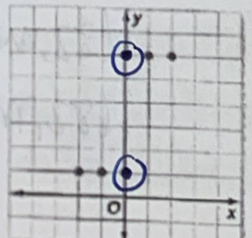
Function



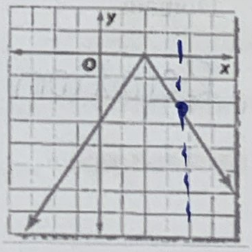
Function



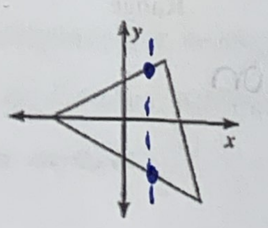
Relation



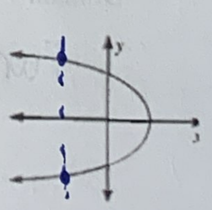
Relation



Function



Relation



Relation

pe 4: Functions Represented Algebraically

Steps:

1.) Get y by itself (isolate y)

2.) Determine if every x will only be paired with one y.

$y=3x-1$

Function ✓

$-2=x^2-4x-y$

$+2 \qquad +y$

$0=x^2-4x+2$

Function ✓

$5x-y=10$

$-5x \qquad -5x$

$\frac{-y}{-1}=\frac{-5x+10}{-1}$

$y=5x-10$ Function ✓

$x+y^2=5$

$-x \qquad -x$

$\sqrt{y^2}=\sqrt{5-x}$

$y=\pm\sqrt{5-x}$

Relation X

$x^2+y=4$

$-x^2 \qquad -x^2$

$y=4-x^2$

Function ✓

$x^3+y=8$

$-x^3 \qquad -x^3$

$y=-x^3+8$

Function ✓

→HW part5

❖ Function Notation

When an equation represents a function, the function is often named by a letter, such as f, g, H. Any letter can be used to name a function. Think of the domain as the set of the function's outputs and the range as the function's inputs. If the function is named *f*, and its input is represented by **x**, then the output is represented by ***f*(x)**. The special notation ***f*(x)**, read "f of x," represents the value of the function evaluated at the number X.

Example: $f(x)=x+5$ x = input values $f(x)$ = output values

input

↑

Y

Input	Function Notation	Output	Ordered Pair
0	$f(0)=0+5$	$f(0)=5$	$(0,5)$
-3	$f(-3)=-3+5$	$f(-3)=2$	$(-3,2)$
7	$f(7)=7+5$	$f(7)=12$	$(7,12)$
1.5	$f(1.5)=1.5+5$	$f(1.5)=6.5$	$(1.5,6.5)$

Examples: Given the following functions, find the following.

If your input includes an 'x', your output will also include an 'x' ↓

1.) $f(x)=x^2+3x+5$

a.) $f(2)$

$f(2)=(2)^2+3(2)+5$

$=4+6+5$

$=15$ $\boxed{(2,15)}$

b.) $f(x+3)$

$f(x+3)=(x+3)^2+3(x+3)+5$

$=(x+3)(x+3)+3x+9+5$

$=x^2+6x+9+3x+14$

$=\boxed{x^2+9x+23}$

c.) $f(-x)$

$f(-x)=(-x)^2+3(-x)+5$

$=\boxed{x^2-3x+5}$

2.) $f(x)=\sqrt{x+1}-5$

a.) $f(2)$

$f(2)=\sqrt{2+1}-5$

$=\boxed{\sqrt{3}-5}$

b.) $f(80)$

$f(80)=\sqrt{80+1}-5$

$=\sqrt{81}-5=9-5=4$

$\boxed{(80,4)}$

c.) $f(x-1)$

$f(x-1)=\sqrt{x\cancel{-1}\cancel{+1}}-5$

$=\boxed{\sqrt{x}-5}$

3.) Evaluate the function $f(x)=\frac{3x^2-1}{x^2}$ at the given values of independent variable and simplify.

a.) $f(-3)$

$f(-3)=\frac{3(-3)^2-1}{(-3)^2}=\frac{3(9)-1}{9}$

$=\frac{27-1}{9}=\boxed{\frac{26}{9}}$

b.) $f(-x)$

$f(-x)=\frac{3(-x)^2-1}{(-x)^2}$

$=\boxed{\frac{3x^2-1}{x^2}}$ → The x^2's do not cancel out!

c.) $f(1)+f(2)$

$f(1)=\frac{3(1)^2-1}{(1)^2}=\frac{3-1}{1}=\boxed{2}$

$f(2)=\frac{3(2)^2-1}{(2)^2}=\frac{3(4)-1}{4}=\frac{11}{4}$

$=\frac{2}{1}+\frac{11}{4}=\frac{8}{4}+\frac{11}{4}=\boxed{\frac{19}{4}}$

→HW part 6

Finding Values for which $f(x)=0$

Ex.) Find all values of x such that $f(x)=0$.

$f(x)=7x-2$

$0=7x-2$
$+2 \quad +2$

$\frac{2}{7}=\frac{7x}{7}$ $x=\frac{2}{7}$

Input = X Output = 0

$\boxed{x=2/7}$

Ex.) Find all values of x such that $f(x)=0$.

$f(x)=x^2-9$

$0=x^2-9$
$+9 \quad +9$

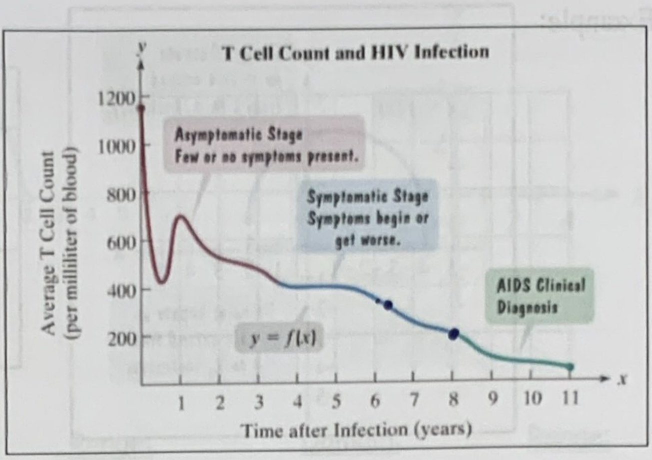
$\sqrt{9}=\sqrt{x^2}$ $x=\pm 3$

Input = X Output = 0

$\boxed{x=\pm 3}$

→HW part 7

❖ Obtaining Information from Graphs



a.) Explain why f represents the graph of a function.

The graph passes the vertical line test.

b.) Use the graph to find and interpret f(8).

$f(8)=200$ After 8 years, the average cell count was 200mm of blood.

c.) For what value of x is f(x) = 350?

$f(x)=350$ when $x=6.5$

❖ Domain and Range of Graphs

1. Square Brackets: indicate endpoints that ARE included in the interval — Example: [-1, 6]

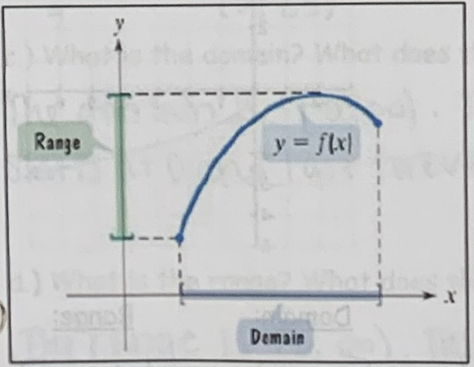
2. Parentheses: indicate endpoints that ARE NOT included in the interval — Example: (-7, 4)

3. One of Each: means that one of the endpoints IS included and the other endpoint IS NOT included — Example: [0, 6)

4. Infinity $-\infty$ or ∞: ALWAYS use parentheses! — Example: $(-\infty, 4)$

   a.) ∞ never ends! Therefore, there is no endpoint.

   b.) $(-\infty,\infty)$ is read as "negative infinity to positive infinity"



Domain: Read from left to right

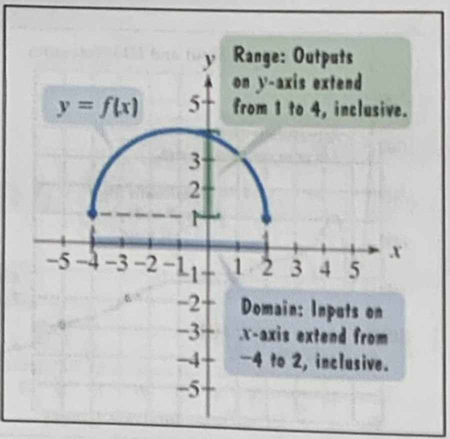
The furthest point included on the left to the furthest point included on the right.

Range: Read from bottom to top

The furthest point at the bottom to the furthest point at the top.

There are two ways to write a function's domain and range: interval notation and set builder notatio

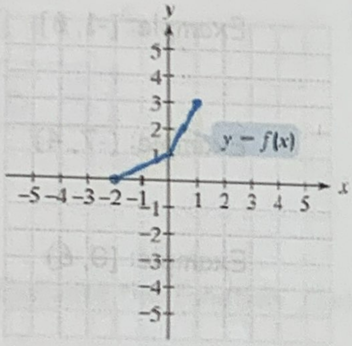
Example:



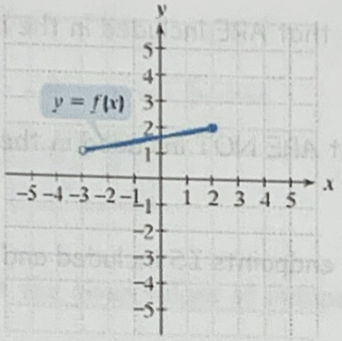
	Interval Notation	Set Builder
Domain	$[-4,2]$	$\{x \mid -4 \le x \le 2\}$
Range	$[1,4]$	$\{y \mid 1 \le y \le 4\}$

Examples: Use the graph of each function to identify its domain and range. Write in interval notation.

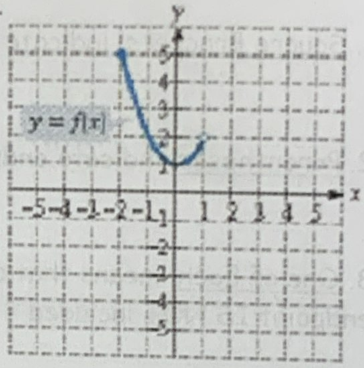
a.



b.



c.

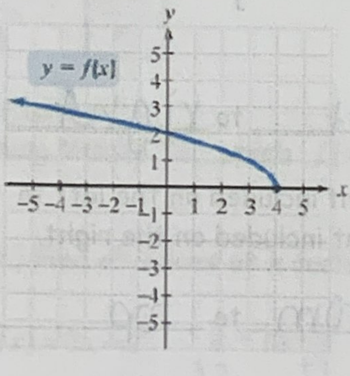


Domain: $[-2,1]$ **Range:** $[0,3]$

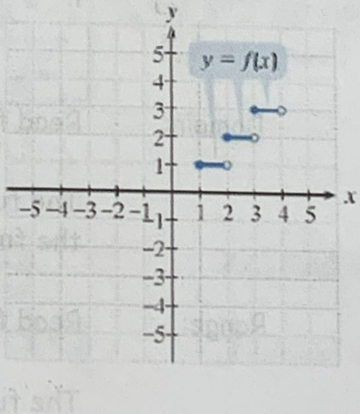
Domain: $(-3,2]$ **Range:** $(1,2]$

Domain: $[-2,1)$ **Range:** $[1,5]$

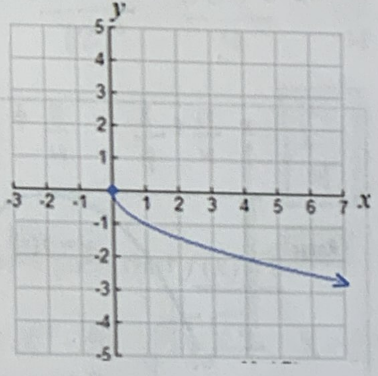
d.



e.



f.



Domain: $(-\infty,4]$ **Range:** $[0,\infty)$

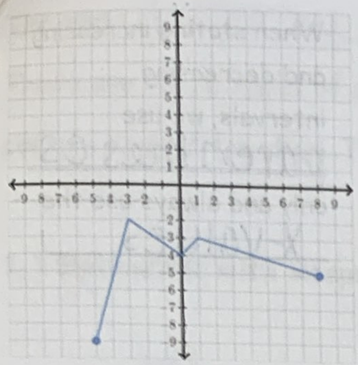
Domain: $[1,4)$ **Range:** $\{1,2,3\}$

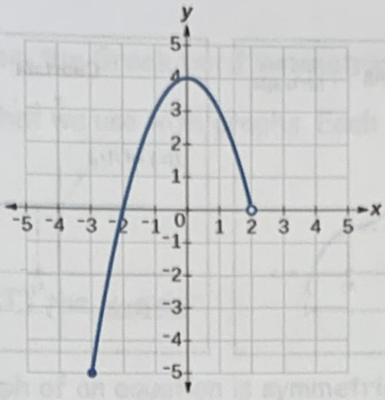
↓

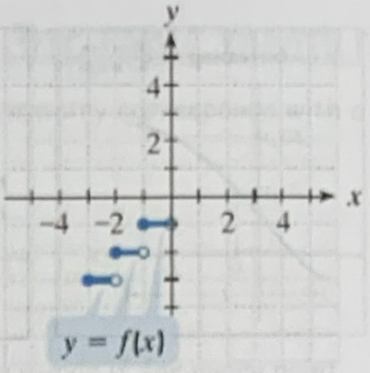
The decimals between are not included in the range

Domain: $[0,\infty)$ **Range:** $(-\infty,0]$

u try!





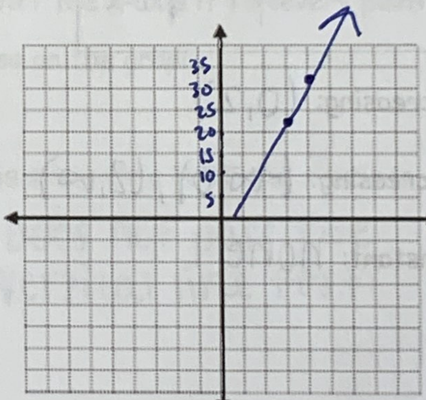


Domain:	Range:	Domain:	Range:	Domain:	Range:
[-5, 8]	[-9, -2]	[-3, 2)	[-5, 4]	[-3, 0)	{-3, -2, -1}

→HW part 8

❖ Analyzing Functions

Swine flu is attacking Pittsburgh. The function P(t) = 9t – 4, determines how many people have swine flu where t = time in days and P = the number of people in thousands. When you have enough information, graph the function.



a.) Find P(4). What does P(4) mean?

$P(4) = 9(4) - 4$

$= 36 - 4$

$= 32$

After 4 days, 32,000 people in Pittsburgh will have the swine flu.

(4, 32)

b.) Find P(t) = 23. What does P(t) = 23 mean?

$23 = 9t - 4$

$+4 \quad +4$

$\frac{27}{9} = \frac{9t}{9} \quad t = 3$

After 3 days, 23,000 people in Pittsburgh will have the swine flu.

(3, 23)

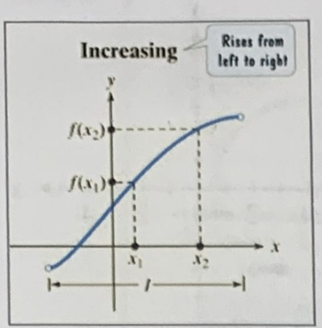
c.) What is the domain? What does the domain mean in this situation?

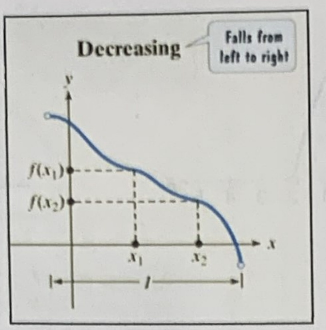
The domain is $[0, \infty)$. The days someone can get the swine flu starts at 0 and last forever.

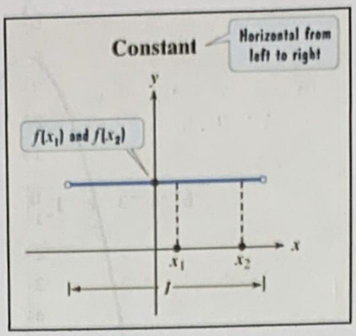
d.) What is the range? What does the range mean in this situation?

The range is $[0, \infty)$. The number of people with swine flu starts at 0 and can infect an infinite number of people.

❖ Increasing and Decreasing Functions

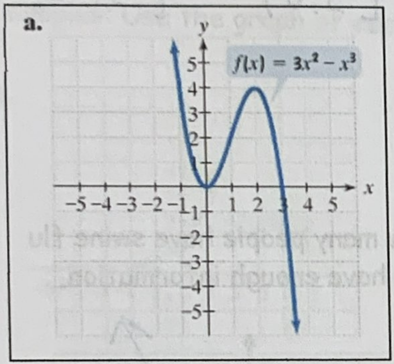


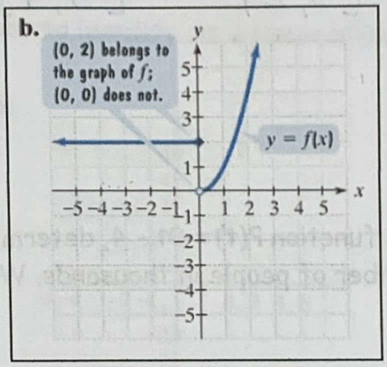


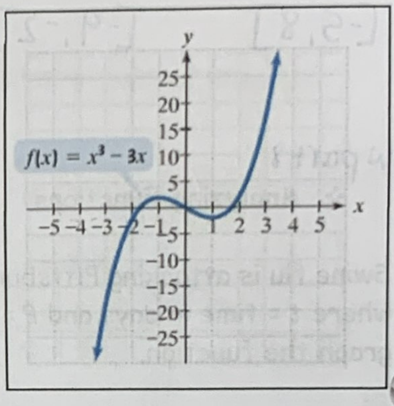


When stating increasing and decreasing intervals, we use parentheses only and always use the X-values!

Examples: State the intervals on which each given function is increasing, decreasing, or constant.







Increasing: $(0,2)$

Decreasing: $(-\infty,0), (2,\infty)$

Constant: none

Increasing: $(0,\infty)$

Decreasing: none

Constant: $(-\infty,0)$

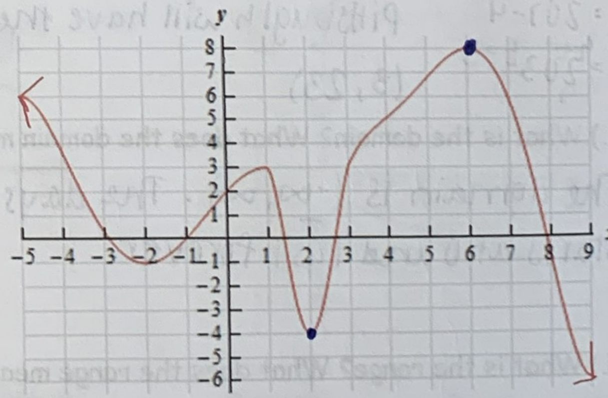
Increasing: $(-\infty,-1), (1,\infty)$

Decreasing: $(-1,1)$

Constant: none

❖ Relative Maxima and Relative Minima

A relative maximum/minimum, also known as a local maximum/minimum is defined as the value of a function, within the domain, which is greater than (max) or less than (min) all of the other points in the *vicinity of the graph.* You write the highest (or lowest) point that you can see!!



Use the graph above...

Relative Max: $(6,8)$

Relative Min: $(2,-4)$

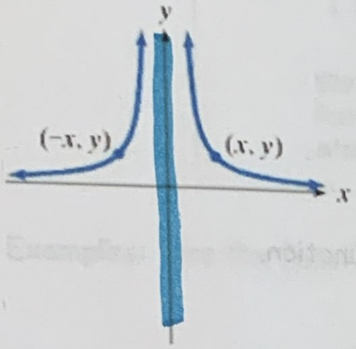
→HW part 9

❖ Symmetry/Even and Odd Functions

The word Symmetry comes from the Greek word symmetria, meaning the same measure

There are three types of symmetry that we use with graphs. Each type of symmetry corresponds with a certain kind of function. They are:

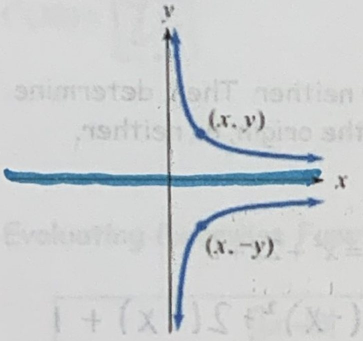
1.) Symmetric with Respect to (WRT) the y-axis:



The graph of an equation is symmetric WRT the y-axis if for every point (x, y) on the graph, the point (-x, y) is also on the graph.

When an equation is symmetric WRT the y-axis, it is also considered to be an even function.

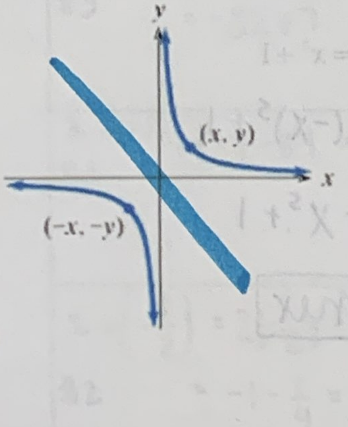
2.) Symmetric with Respect to (WRT) the x-axis:



The graph of an equation is symmetric WRT the x-axis if for every point (x, y) on the graph, the point (x, -y) is also on the graph.

Typically, the graph of an equation that is symmetric WRT the x-axis is not considered to be a function. Why ? Does not pass the vertical line test

3.) Symmetric with Respect to (WRT) the origin:

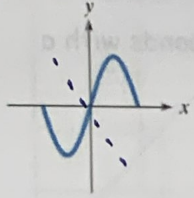


The graph of an equation is symmetric WRT the origin if for every point (x, y) on the graph, the point (-x, -y) is also on the graph.

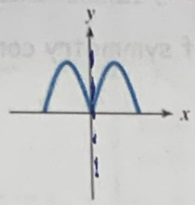
When an equation is symmetric WRT the origin, it is also considered to be an odd function.

*Think of the graph flipping 2 times. This means it's symmetric WRT the origin.

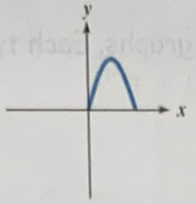
Examples: Determine whether each graph is the graph of an even function, an odd function, or a function that is neither even or odd. For the ones that are neither, explain why.

a.


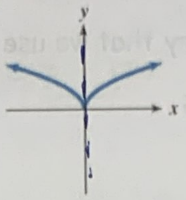
Odd

b.


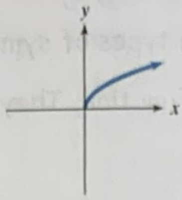
even

c.


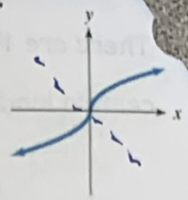
none

a.


even

b.


none

c.


odd

Finding Even/Odd Functions Using an Equation Only

To test for symmetry, substitute $-x$ in for x and simplify.

- If the equation stays the exact same, the function is an even function.
- If the equation all changes, the function is an Odd function.
- If some of the equation stays the same and some of the equation changes, then the function is neither an even or odd function.

Examples: Determine whether each of the following functions is even, odd, or neither. Then, determine whether the function's graph is symmetric with respect to the y-axis, x-axis, the origin, or neither. (We can check our answers graphing on Desmos.com)

a.) $f(x) = x^3 - 6x$

$f(-x) = (-x)^3 - 6(-x)$

$= -x^3 + 6x$

$\boxed{\text{odd}}$

b.) $g(x) = x^4 - 2x^2$

$g(-x) = (-x)^4 - 2(-x)^2$

$= x^4 - 2x^2$

$\boxed{\text{even}}$

c.) $h(x) = x^2 + 2x + 1$

$h(-x) = (-x)^2 + 2(-x) + 1$

$= x^2 - 2x + 1$

$\boxed{\text{Neither}}$

d.) $f(x) = x^2 + 6$

$f(-x) = (-x)^2 + 6$

$= x^2 + 6$

$\boxed{\text{Even}}$

e.) $g(x) = 7x^3 - x$

$g(-x) = 7(-x)^3 - (-x)$

$= -7x^3 + x$

$\boxed{\text{odd}}$

f.) $h(x) = x^5 + 1$

$h(-x) = (-x)^5 + 1$

$= -x^5 + 1$

$\boxed{\text{Neither}}$

*See any patterns? LOOK at the powers!

→HW part 10

⁖ Evaluating Piecewise Functions

A function that is defined by two (or more) equations over a specified domain is called a piecewise function

$$C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \\ 20 + 0.40(t - 60) & \text{if } t > 60 \end{cases}$$

The cost is \$20 for up to and including 60 calling minutes.

The cost is \$20 plus \$0.40 per minute for additional time for more than 60 calling minutes.

\$20 for first 60 minutes | **\$0.40 per minute** | **times the number of calling minutes exceeding 60**

Examples: Use the function that defines the telephone plan to find and interpret each of the following:

$$C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \rightarrow \text{Between 0 and 60} \\ 20 + 0.40(t - 60) & \text{if } t > 60 \rightarrow \text{greater than 60} \end{cases}$$

$C(30) = \boxed{20}$

$C(100) =$
$= 20 + 0.40(100-60)$
$= 20 + 0.4(40) = 20 + 16$
$= \boxed{36}$

$C(80) =$
$= 20 + 0.40(80-60)$
$= 20 + 0.4(20) = 20 + 8$
$= \boxed{28}$

Evaluating Piecewise Functions – Based on the given piecewise function, evaluate the following.

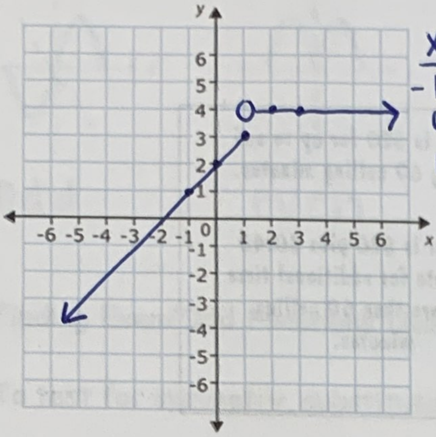
Given $a(x) = \begin{cases} |x-8| & \text{if } x \le -6 \\ 2x - x^2 & \text{if } -6 < x \le 1, \\ -4x+7 & \text{if } x > 1 \end{cases}$ **find each function value.**

1. $a(8)$ $= -4(8)+7$ #3 $= -32+7 = \boxed{-25}$	2. $a(1)$ $= 2(1) - (1)^2$ #2 $= 2 - 1 = \boxed{1}$
3. $a(-7) = \lvert -7-8 \rvert = \lvert -15 \rvert$ #1 $= \boxed{15}$	4. $a(-3)$ $= 2(-3) - (-3)^2$ #2 $= -6 - 9 = \boxed{-15}$
5. $a\left(-\frac{1}{2}\right) = 2(-\frac{1}{2}) - (-\frac{1}{2})^2$ #2 $= -1 - \frac{1}{4} = \boxed{-5/4}$	6. $a\left(\frac{9}{4}\right) \rightarrow a(2\,1/4)$ $= -4(\frac{9}{4}) + 7$ #3 $= -9 + 7 = \boxed{-2}$

→ HW part II

Graph the piecewise functions defined by:

$$f(x) = \begin{cases} x + 2 & \text{if } x \le 1 \\ 4 & \text{if } x > 1. \end{cases}$$



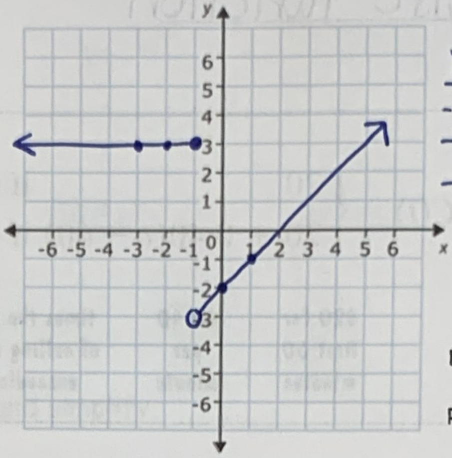
X	Y
-1	1 ↓
0	2
1	3 •

X	Y
1	4 o
2	4
3	4 ↓

Domain: (-∞, ∞)

Range: (-∞,3] ∪ {4}

$$f(x) = \begin{cases} 3 & \text{if } x \le -1 \\ x - 2 & \text{if } x > -1. \end{cases}$$



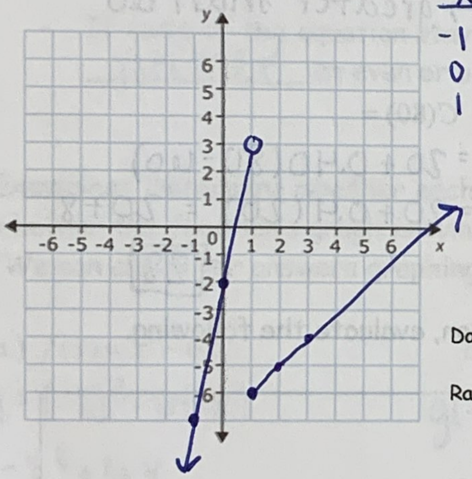
X	Y
-3	3 ↑
-2	3
-1	3 •

X	Y
-1	-3 o
0	-2
1	-1 ↓

Domain: (-∞, ∞)

Range: (-3, ∞)

$$f(x) = \begin{cases} 5x - 2 & \text{if } x < 1 \\ x - 7 & \text{if } x \ge 1 \end{cases}$$



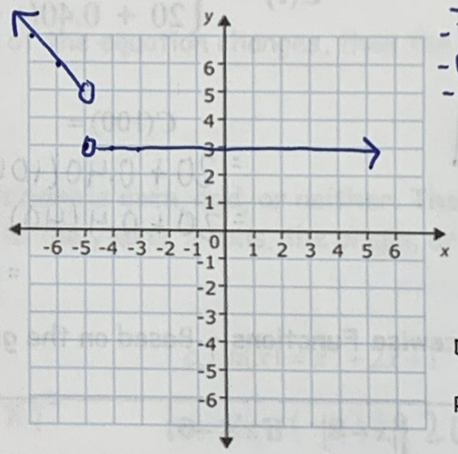
X	Y
-1	-7 ↑
0	-2
1	3 o

X	Y
1	-6 •
2	-5
3	-4 ↓

Domain: (-∞, ∞)

Range: (-∞, ∞)

$$g(x) = \begin{cases} -x & \text{if } x < -5 \\ 3 & \text{if } x > -5 \end{cases}$$



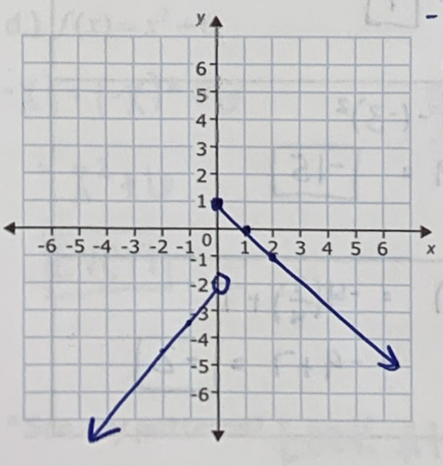
X	Y
-7	7 ↑
-6	6
-5	5 o

X	Y
-5	3 o
-4	3
-3	3 ↓

Domain: (-∞,-5) ∪ (-5,∞)

Range: {3} ∪ (5,∞)

$$h(x) = \begin{cases} \frac{4}{3}x - 2 & \text{if } x < 0 \\ -x + 1 & \text{if } x \ge 0 \end{cases}$$



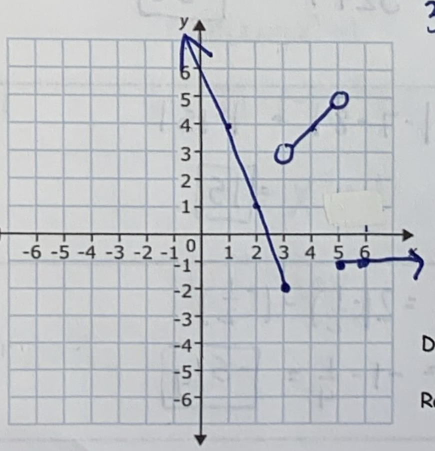
X	Y
-2	-4.6 ↑
-1	-3.3
0	-2 o

X	Y
0	1 •
1	0
2	-1

Domain: (-∞, ∞)

Range: (-∞, 1]

$$p(x) = \begin{cases} -3x + 7 & \text{if } x \le 3 \\ x & \text{if } 3 < x < 5 \\ -1 & \text{if } x \ge 5 \end{cases}$$



X	Y
1	4 ↑
2	1
3	-2 •

X	Y
3	3 o
4	4
5	5 o

X	Y
5	-1 •
6	-1
7	-1 ↑

Domain: (-∞, ∞)

Range: [-2, ∞)

→HW part 12